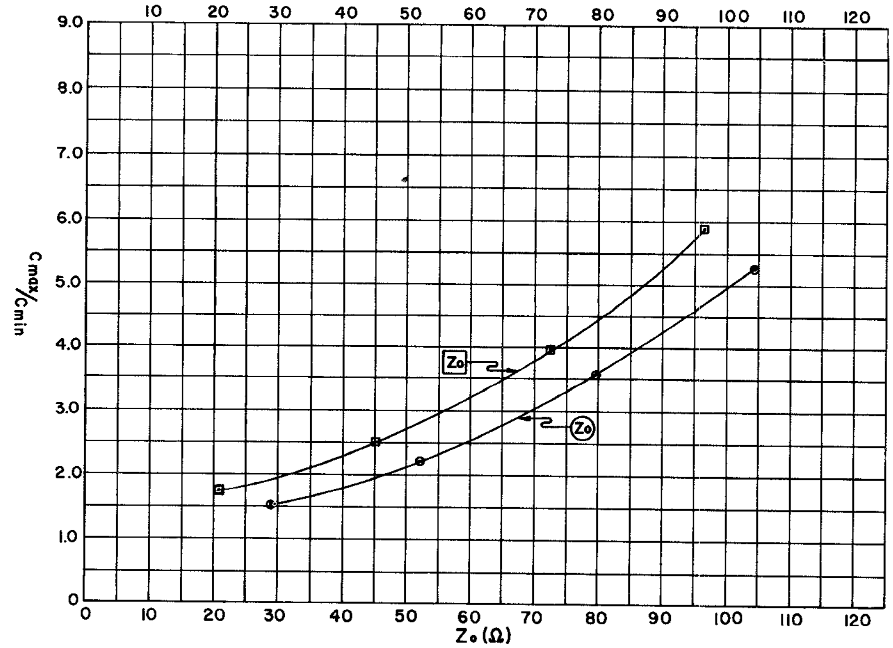

Fig. 2. Characteristic impedances.

relative shape of this rectangle which, in turn, determines whether the inner and outer conductors of the two coaxial systems will be close together or far apart. As $k \to 0$, side $AB \to \infty$, and both characteristic impedances $\to \infty$; while as $k \to 1$, side $0A \to \infty$ and both characteristic impedances $\to 0$. For the curves of Fig. 2, k varies from 0.005 to 0.65.

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Comments on "Transmission Line Impedance Matching Using the Smith Chart"

P. I. SOMLO

For the recently discussed problem of R. M. Arnold[1] of finding the characteristic impedance and electrical length of a uniform, lossfree transmission line which transforms a given impedance into another given impedance, an alternate approach using an elementary auxiliary calculation (with no trial and error) has been given in [1].

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An Alternate Derivation and Procedure for Cristal's Transformation for Transmission-Line Networks

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Abstract—**This letter gives an alternate derivation and procedure for Cristal's transformation which transforms commensurate transmission-line networks with unit elements from the frequency variable λ to $1/\lambda$, where $\lambda = \tanh(\gamma L)$.**

Given a network with input impedance $Z(\lambda)$ composed of commensurate transmission-line elements including unit elements, we will now give a procedure for finding a network with input impedance $Z(1/\lambda)$ directly from the element values of the given circuit. This derivation is believed to be simpler, the method easier to apply and somewhat more general.

The method is based on the following theorem: If the network of Fig. 1(a) has input impedance $Z(\lambda)$ then the input impedance of the circuit of Fig. 1(b) is $Z(1/\lambda)$.

Proof: The input impedance of the network of Fig. 1(a) is given by

$$Z_A = Z_0 \frac{Z_L(\lambda) + \lambda Z_0}{Z_0 + \lambda Z_L(\lambda)}. \tag{1a}$$

Since the unit element in Fig. 1(b) is terminated by a load impedance $Z_0^2/Z_L(1/\lambda)$, the input impedance of the network of Fig. 1(b) is given by

$$Z_B = Z_0 \frac{Z_0^2/Z_L(1/\lambda) + \lambda Z_0}{Z_0 + \lambda[Z_0^2/Z_L(1/\lambda)]} = Z_0 \frac{Z_L(1/\lambda) + (1/\lambda)Z_0}{Z_0 + (1/\lambda)Z_L(1/\lambda)}. \tag{1b}$$

Comparison of (1a) and (1b) shows that $Z_B = Z_A(1/\lambda)$. Q.E.D. Using the above theorem we can easily transform ladder networks containing unit elements from the λ to the $1/\lambda$ plane. Consider as an example the network of Fig. 2(a) with input impedance $Z(\lambda)$. Using the above theorem we obtain the network of Fig. 2(b) with input impedance $Z(1/\lambda)$. Eliminating the impedance inverter then

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